KINEMATIC SIMILARITY OF A TURBULENT SWIRLED FLOW IN A PIPE

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It is shown that in the case of a developed turbulent rotating flow in a pipe kinematic similarity occurs at large distances from the swirl source.

It was found in [1] that turbulent rotating flows in pipes with swirling inserts located over the entire length of the channel are similar, and the intensity of the heat- and mass-transfer processes in them is determined by the value of the effective Reynolds number: $Re_* = Re\sqrt{1+\tan^2\alpha}$, where α is the helical angle of the insert.

If the swirl source is located only near the channel entrance and not along its entire length, rotation is damped as the liquid moves along the pipe and the problem of similarity of the flow is not evident.

During flow with large Reynolds numbers in a pipe, let a fully developed turbulent flow be established at some sufficiently remote distance from the swirler. In this case, in the φ component of the Reynolds equations for an axisymmetric flow

$$V_x\frac{\partial V_\varphi}{\partial x}+V_r\frac{\partial V_\varphi}{\partial r}+\frac{V_rV_\varphi}{r}=\nu\left(\frac{\partial^2V_\varphi}{\partial x^2}+\frac{\partial^2V_\varphi}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial V_\varphi}{\partial r}-\frac{V_\varphi}{r^2}\right)-\left(\frac{\partial}{\partial x}\overline{uw}+\frac{\partial}{\partial r}\overline{vw}+2\frac{\overline{vw}}{r}\right) \tag{1}$$

(V_x, V_r, V_φ are the values of the average velocities in axial, radial, and tangential directions; u, v, w are their fluctuations in a turbulent flow) we can neglect the second and third terms in the left side and by analogy with the case of one-dimensional flow, the quantity $\partial(\overline{uw})/\partial x$ in its right side. Then for a liquid flow outside the region of the viscous sublayer on the pipe walls we will have

$$V_x\frac{\partial V_\varphi}{\partial x}=-\left(\frac{\partial}{\partial r}\overline{vw}+2\frac{\overline{vw}}{r}\right). \tag{2}$$

Using the representation of Reynolds stresses caused by turbulent fluctuations

$$\rho\overline{vw}=-\varepsilon\left(\frac{\partial V_\varphi}{\partial r}-\frac{V_\varphi}{r}\right) \tag{3}$$

(ε is the turbulent exchange coefficient, which in the first approximation can be considered independent of r [2]) and changing to dimensionless variables

$$W=\frac{V_\varphi}{V_0},\quad U=\frac{V_x}{V_0},\quad Re=\frac{V_0r_0}{\nu},\quad \xi=\frac{r}{r_0},\quad \zeta=\frac{x}{r_0}, \tag{4}$$

where V_0 is the maximum value of V_x and r_0 is the pipe radius, we obtain

$$\frac{\partial^2W}{\partial\xi^2}+\frac{1}{\xi}\cdot\frac{\partial W}{\partial\xi}-\frac{W}{\xi^2}=Re\frac{\rho\nu}{\varepsilon}U\frac{\partial W}{\partial\zeta}. \tag{5}$$

We will determine what the rotational velocity profile should be in the flow being considered if kinematic similarity of the flow occurs, i.e., $W=\omega(\xi)W_m(\zeta)$ ($W_m=V_{\varphi max}/V_0$ is the maximum value of the relative tangential velocity in a given section of the pipe). It is easy to see that in this case Eq. (5) is transformed to

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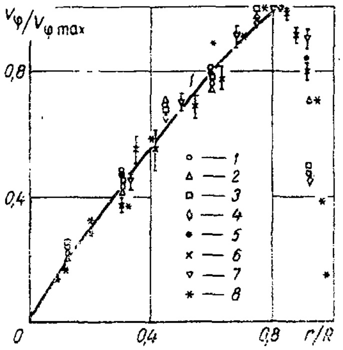

Fig. 1

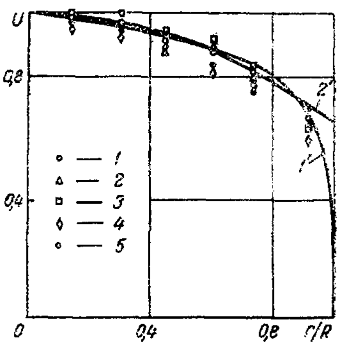

Fig. 2

Fig. 1. Similar rotational velocity profile $V_\varphi/V_{\varphi\max}$ in pipe with a band swirler at the entrance ($\tan\alpha$ = 1): 1) L/d = 5; 2) 14; 3) 23; 4) 32; 5) 47; 6) according to data in [4]; 7) [5]; 8) [3].

Fig. 2. Similar axial velocity profile $U = V_x/V_0$ in pipe with band swirler at entrance ($\tan\alpha$ = 1): L/d = 5; 2) 14; 3) 23; 4) 32; 5) 47.

$$\frac{\dfrac{d^2\omega}{d\xi^2}+\dfrac{1}{\xi}\cdot\dfrac{d\omega}{d\xi}-\dfrac{\omega}{\xi^2}}{\omega U} = \mathrm{Re}\frac{\rho\nu}{\varepsilon}\cdot\frac{1}{W_m}\cdot\frac{dW_m}{d\zeta} = -\lambda^2. \tag{6}$$

Hence for $\omega(\xi)$ we have

$$\xi^2\ddot{\omega}+\xi\dot{\omega}+\omega(\lambda^2U\xi^2-1)=0\,, \tag{7}$$

and for the maximum relative rotational velocity

$$W_m = A\exp\left(-\frac{\lambda^2}{\mathrm{Re}}\cdot\frac{\varepsilon}{\rho\nu}\,\zeta\right). \tag{8}$$

In a steady rotating flow U ≈ 1 near the pipe axis, and therefore from (7) follows

$$\omega = C_0J_1(\lambda\xi)+C_1Y_1(\lambda\xi). \tag{9}$$

This solution can be refined if necessary by using the actual relation $U(\xi)$ in a swirled flow. For flow outside a viscous sublayer the boundary conditions of Eq. (7) will be $\omega = 0$ for $\xi = 0$ and $\omega = 1$ for $\xi = \xi_1$ (ξ_1 is the distance from the axis to the place where $V_\varphi = V_{\varphi\max}$). Then the expected similar rotational velocity profile in a developed turbulent flow in the first approximation should have the form

$$\frac{V_\varphi}{V_{\varphi\max}} = \frac{J_1(\lambda\xi)}{J_1(\lambda\xi_1)}. \tag{10}$$

It is natural that such an expression will not reflect the actual picture of flow in the immediate vicinity of the walls, where the effect of molecular viscosity is considerable.

A series of experiments was undertaken to check the conclusion made concerning the possibility of the existence of kinematic similarity of a rotating flow. By means of miniature pressure probes (diameter about 1 mm) we investigated the velocity and pressure fields in a flow of water with Re = $5\cdot10^3$–$5\cdot10^4$ in a glass pipe with a 27 mm diameter. A twisted band or an endless screw with different helical angles was used as the swirling inserts at the pipe entrance. Figure 1 shows the results of measuring the relative rotational velocity of the liquid obtained at different distances from the twisted band with α = 45°. (Two loops of the helix were placed over the length of the swirler.) We see that for the type of swirler used, as early as beginning with L/d = 5, the tangential velocity profile, at least in the first approximation, is similar and it can be described by the curve in Fig. 1. The maximum value of the rotational velocity occurs approximately at r/r_0 = 0.80. Data of other known measurements of the profile $V_\varphi/V_{\varphi\max}$ [3-5] are also plotted in Fig. 1. As we see, these data do not contradict the results obtained here. In connection with Eq. (1) for the rotational velocity in a turbulent flow we should note that an analogous form of the profile is obtained on the assumption of a helical character of flow in the pipe [6, 7]. Such an agreement indicates that the turbulent swirled flow of liquid in the greater part of the pipe cross section is actually close to helical and the vortex lines in it coincide with the streamlines. The establishment of this fact confirms the possibility of using a model of a helical flow of an ideal liquid for calculating certain effects of real rotating flows [7, 8].

Figure 2 presents the results of measuring the distribution of the relative axial velocity. Curve 1 corresponds to the "1/7 power" law, which usually occurs in the case of a one-dimensional flow in a pipe, and curve 2 shows the distribution of the axial velocity in a helical flow $U = J_0(1.20\xi)$. It is of interest to determine the rate of damping of the swirl over the length of the pipe. Experimental points describing $V_{\varphi max}/V_{\varphi 0}$ as a function of L/d ($V_{\varphi 0}$ is the value of $V_{\varphi max}$ at the exit of the swirler) are given in Fig. 3 in semilogarithmic coordinates. The results of a series of experiments conducted during flow of a rotating gas flow ($T \approx 1500$°K, $\mu = 5 \cdot 10^{-4}$ P) are also presented there along with the data obtained by means of the method described above for flow of water. Damping of the swirl in these experiments was determined from the angle of slope of the wake of the helical streamline on the smoked wall of the pipe.

In conformity with Eq. (8) the experimental points (Fig. 3) lead to a value of the turbulent exchange coefficient in swirled flows equal to

$$\varepsilon/\rho\nu = 6 \cdot 10^{-3}\,\mathrm{Re}. \tag{11}$$

Thus the rate of damping of the swirl over the pipe length in the case of using swirling inserts in the form of a twisted band does not depend on the initial intensity of rotation and can be described approximately by the formula

$$\frac{V_{\varphi\max}}{V_{\varphi 0}} = \exp\left(-1.7 \cdot 10^{-2}\frac{L}{d}\right). \tag{12}$$

The experiments show that the level of the transitional section beyond which self-preservation of the velocity profile occurs depends not only on the magnitude of the initial swirling of the flow but also on the design of the swirler (endless screw, twisted band, tangential inlet, endless screw with a central hole, etc.). Figure 4 presents, for an example, the data on the relative axial and tangential velocity profiles of flow in a pipe when a double endless screw with a length of five pitches and $\tan\alpha = 2.0$ was used as the swirler. It is seen that in the case of a large magnitude of swirling of the flow at the channel entrance some slowing of the longitudinal flow of the liquid occurs near the flow axis (even the formation of a "bubble" of back currents is possible near the swirler). The length of the transitional section needed for stabilizing the turbulent flow in a pipe beyond the screw swirler in the case of strong rotation is, as follows from the experiments, about 40-50 diameters. A decrease of $\tan\alpha$ of the swirler shortens the section of flow

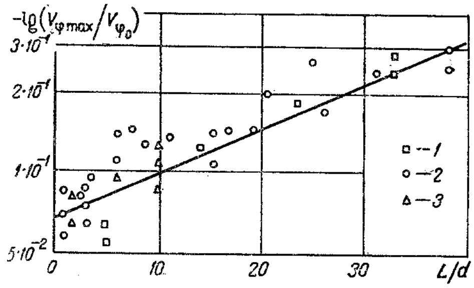

Fig. 3. Damping of the swirl over the pipe length: 1) water, d = 27 mm, Re = $5 \cdot 10^3$; 2) gas, d = 20 mm, Re = $2.5 \cdot 10^5$; 3) gas, d = 14 mm, Re = $1 \cdot 10^4$.

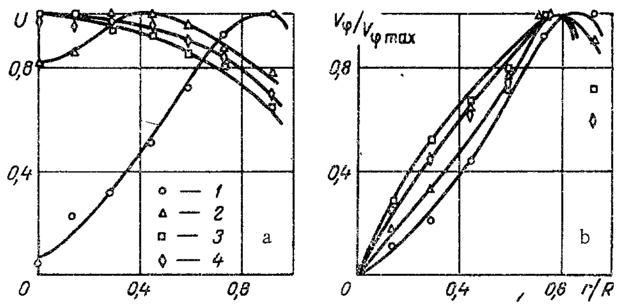

Fig. 4. Dimensionless profile: a) axial velocity in pipe with endless screw $\tan\alpha = 2$ and b) for rotational velocity: 1) L/d = 3; 2) 20; 3) 30; 4) 45.

stabilization. In this case the relative axial velocity profile experiences an especially marked change in the acceleration section; the rotational velocity profile changes much less.

NOTATION

V_x, V_r, V_φ	are the axial, radial, and tangential velocity components of flow;
x	is the distance along pipe axis;
r, r_0	are the current value of radius and pipe radius;
ν	is the viscosity coefficient;
ε	is the turbulent exchange coefficient;
$Re = V_0 r_0/\nu$	is the Reynolds number;
ξ, ζ	are the dimensionless radius and length.

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